An Historical Perspective

The written history of the Platonic Solids begins, appropriately, with Plato (c. 427-347 B.C.). In his Timaeus, he describes the five regular polyhedra both geometrically and cosmologically. Archimedes (c.287-212 B.C.), the Greek mathematician from Syracuse, was the first to give a written account of the thirteen semi-regular polyhedra; and hence their being referred to as Archimedean Solids.

Other major contributors to this body of knowledge include: Euclid (c.300 B.C.) the great geometrician from Alexandria; Heron (c.62 A.D.) the Greek mathematician; Ptolemy (c.150 A.D.) the Greek astronomer; the unknown author of Euclid XV (c.300 A.D.); Al-Kindi (794-874) the earliest major Arab philosopher; Al-Biruni (973-1050) arguably the most influential scientist of his time; and among the Europeans, Campanas of Novara, during the thirteenth century, and Maurolyeus (1494-1575).

Johann Kepler (1571-1630), the great German astronomer and scientist, employed an understanding of the reciprocal nature of the five regular polyhedra in his famous theory of planetary rotation. Rene Descartes (1596-1650) was also interested in polyhedra, and around 1635 discovered what was later to become known as Euler's Theorem for polyhedra: the number of vertices, minus the number of edges, plus the number of faces equals two (V-E+F=2). Descartes' writings on this topic were not published until 1860. Leonhard Euler (1707-1783) arrived at this same formula independently in 1752, and being the first to publish, was given the lasting credit of the formula bearing his name.

More recently, the work of Buckminster Fuller has helped to renew interest in these forms. For example, scientists who recently discovered a new carbon isotope, C60, with the molecular structure of a truncated icosahedron have given this new compound the name of Fullerite, sometimes called the "Bucky Ball." This form is familiar to children around the world as the common soccer ball.

Mystical and Natural

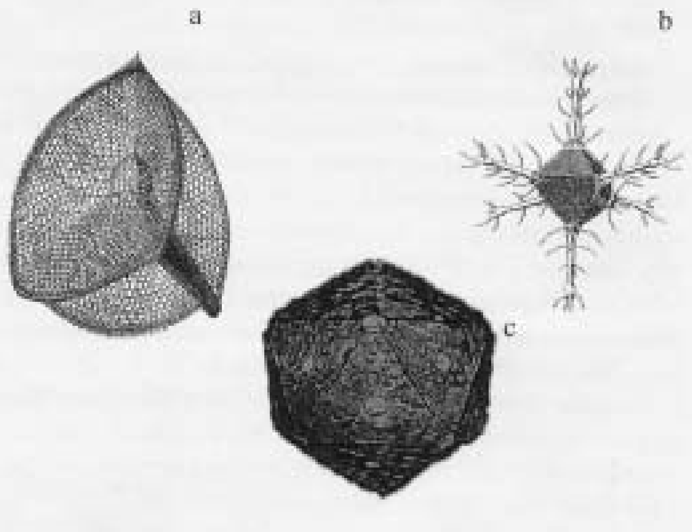
The idea that these shapes are imbued with mystical and cosmological significance appears to be as old as mankind's familiarity with the shapes themselves. Plato ascribes the five solids to the five elements: the cube to earth; the icosahedron to water; the octahedron to air; the tetrahedron to fire; and the dodecahedron to aether.

Al-Biruni attributed the same mystical qualities to the five regular polyhedra as Plato. These ideas were adopted and developed by other Arab philosophers, astronomers and mathematicians, whose works were, in turn, influential in introducing these concepts to Europe. Even Johann Kepler, despite his being regarded as the father of modern astronomy, was not above attributing cosmological and mystical significance to the figures.

As if to confirm the archetypal nature of these beautiful forms, nature herself has not been shy to utilize their inherent structural stability. Atoms will form molecules which conform to polyhedral symmetry. For example, silicon and oxygen ions will always form a tetrahedral structure; and alumina hydrate in the form of gibbsite forms an octahedron. When polyhedral molecules are connected in a close packed three dimensional lattice, crystals are formed.

Within the vast realm of crystallography, some of the regular and semi-regular polyhedra are commonly found. The mineral sodium sulphantimoniate will form tetrahedral crystals; common salt crystals will form a cube; diamonds, spinals and chrome alum will crystallize into octahedrons; pyrite will form into nearly regular dodecahedrons; and fluorite will crystallize into a nearly regular truncated tetrahedron.

Certain viruses will also take on the structure of regular and semi-regular polyhedra, and all five of the Platonic Solids have been found in the plethora of skeletal remains of microscopic Radiolaria plankton. Truly, these pure forms are both beautiful and fundamental to the fabric of creation.



a. Anassellarian skeleton has the geometry of the tetrahedron. b. Skeletal remains of the radiolara C. octahedrus shows an octahedral geometry and c. A virus with an icosahedral geometry.

The Arts

It is impossible to know the extent of knowledge that our early ancestors had of these remarkable and beautiful shapes. One can imagine with what wonder prehistoric people must have regarded the occasional discovery of crystalline examples of these forms. Considering the beauty of the forms, it is not surprising that the Platonic and

Archimedean Solids have been used in the arts. The fact that they have not been used widely only makes those examples we do find all the more interesting.

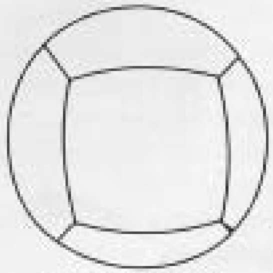
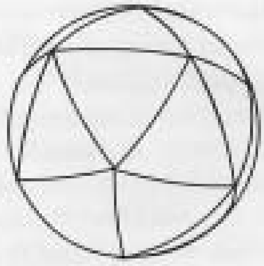
Early Geometric Models

Perhaps the most remarkable ancient examples of regular and semi-regular polyhedra have been found in Scotland and Northern Ireland, and date back to the Neolithic period. The same people who produced the celebrated stone circles also produced numerous examples of carved stone balls which clearly represent the geometry of all five of the Platonic and several of the Archimedean solids.

Late Neolithic carved balls. Photograph courtesy of Ashmolean Museum Oxford, England, Department of Antiquities.

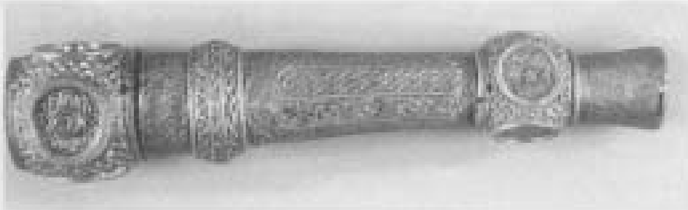
These beautifully carved artifacts from the ancient British Isles were carved in a manner which gives emphasis to their spherical origin, yet is distinctly polyhedral in geometry. They are rather small, fitting comfortably into an adult's hand. Most of them were carved from basalt, a very hard stone, and great care and time went into their fabrication. One can only speculate as to the possible use of these remarkable objects; but without doubt, they must have been of great significance to this culture. They certainly provide clear evidence that these people had a far greater knowledge of geometry than is commonly recognized.

To understand the spherical geometry of these incised stone balls, it is helpful to know the relationship between the polyhedra and sphere. Each of the five Platonic and thirteen Archimedean Solids can be said to be contained within a sphere. The vertices of each rest upon what is referred to as the "great sphere", and are of equal distance from the center. By drawing a line from vertex to vertex along the surface of the sphere, it is therefore possible to give a spherical representation to these polyhedra.

spherical cube . spherical icosahedron

It is interesting that the identical spherical representation of the cube which the Neolithic Celts used on their stone balls was also used well over two thousand years later by Irish Celtic metal workers. An early ninth century silver thistle brooch with characteristic rounded cubic ornamentation was found in Cashel, County Tipperary; and similar ornamental features are found on an eighth century crozier ferrule of gilt copper and amber. In both examples, the rounded areas which would have been the corners and edges of the cube have been filled with an interweaving ornament so characteristic of this rich artistic tradition.



8th century crozier ferrule, Ireland. Photo courtesy of the National Museum of Ireland, Dublin.

But it was the Arabs and Persians who seem to have given the greatest expression to the practical application of the polyhedra forms. The cube and icosahedron, as well as the icosidodecahedron, were used during the twelfth century in the fabrication of several beautiful twisted gold wire and granulated earrings. A remarkably beautiful incense globe from Iran or Afghanistan, circa 1200, is ornamented with six interweaving "great circle" bands which make up the icosidodecahedron in spherical form. Each of the twelve pentagonal "faces" on this globe depict one of the signs of the zodiac.

Persian Incense Globe, Iran or Afganistan c.1200, brass inlaid with silver. Picture from "The Unity of Islamic Art" King Faisal Center for Research and Islamic Studies.

Several Islamic vases, some made from bronze and others from clay, dating as early as the twelfth century, employ a cuboctahedron as the primary form of the vessel. Islamic examples such as these almost certainly inspired the making of similar vases in China. Chinese sculptors occasionally used polyhedral spheres in the well-known lion motif, wherein one paw rests upon a globe. Europe seems to have utilized polyhedral forms less frequently than Islamic or Chinese cultures. Yet both Leonardo da Vinci (1452-1519, Italy) and Albrecht Durer (1471-1528, Germany) were interested in polyhedra and illustrated several examples of stellated forms. Much more recently, in his characteristically ingenious manner, the Dutch artist M.C. Escher (1898-1972) used these forms as the geometric basis for several of his prints and wood carvings.

Islamic vessel with cuboctahedron body. Picture from the Victoria and Albert Museum, South Kensington,

ISLAMIC PATTERNS

The designs used in the ornamentation of the eighteen models in Geodazzlers are characteristic of the tradition of Islamic geometric pattern making. This rich tradition is fundamental to the art and architecture of Islam, and along with floral design and, especially, calligraphy, forms the three qualities which characterizes Islamic art and ornament. In no other culture has the use of ornament received greater attention, or more varied application. In the realm of architecture, other cultural traditions have tended to treat ornament as a form of surface adornment. By contrast, Islamic architecture always seeks to create a unified whole, wherein the ornament is integral with the form itself. The architectural proportions of traditional Islamic buildings are often governed by the strict proportional requirements of a specific geometric pattern.

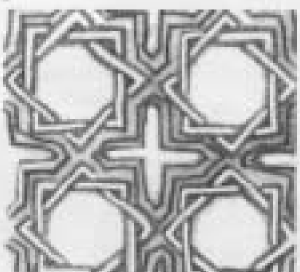
Much has been written about the Islamic interdiction which forbids the representation of nature, and especially the human form, in art. While such restrictions certainly contributed significantly to the development of Islamic art in general, and the emphasis on geometric patterns in particular, it would be a mistake to assume that Islamic artists were in any way restricted in their ability to derive true inspiration, or create works of the very highest degree of artistic expression. Islamic geometric patterns are intrinsically beautiful, and it is not surprising that this tradition continued to develop for so many centuries, and to appeal to so many people from such diverse lands. From Southern Spain to Northern India, from North Africa to Central Asia, this tradition of pattern making has been an essential element to the arts.

As with any great artistic tradition, this geometric ornamentation has developed distinctive stylistic differences over time and as interpreted by the diverse cultures within Islam. And to add to the richness of this tradition, these patterns have found expression in a great variety of materials and techniques. These include carved and inlaid wood, carved stone, ceramic tile, cut-tile mosaic, stained glass, carved gypsum, cast and pierced metal, textiles, illuminated manuscripts and painted miniatures. Truly this is an artistic tradition without limit in scope.

Application of Islamic Pattern to Polyhedra

But why apply Islamic geometric patterns to Platonic and Archimedean solids? Given the knowledge which Islamic artists had of these three dimensional forms, and their strong inclination toward geometric ornament, it is perhaps surprising that this combination was not the focus of some attention. One important exception to the lack of such a combination is found on the famous Janeta sward. This dates from fifteenth century Spain, and belonged to the last Nasrid monarch, Muhammad XII, known as Boabdil. The hilt is ornamented in cloisonne enamel and gilt silver. The Pommel, or end piece, of the hilt is in the form of a small ornamented sphere. The geometric pattern on this Pommel is derived from the square faces of a cubic division of the sphere, and is a simple, yet beautiful example of the traditional "star & cross" pattern so prevalent throughout Islamic lands. However, being that this pattern is placed upon a sphere rather than on a two dimensional surface, the nature of the pattern changes in such a way that the "cross" element takes on a three fold symmetry!

a

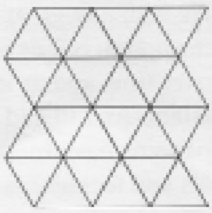


b

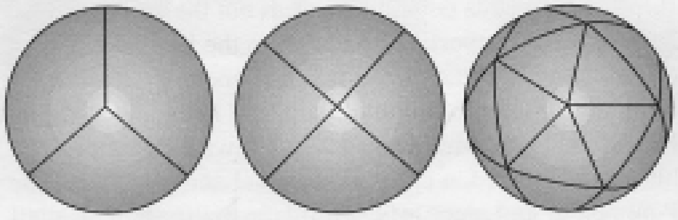
a. A traditional Islamic star and cross pattern. b. Janeta sward Pommel shows how the cross from the traditional star and cross is transformed into a three pointed "cross" due to it being inscribed on the surface of a spherical cube.

The Janeta sward Pommel is an excellent example of how the three dimensional geometry of the regular and semi-regular polyhedra create unique opportunities for design. Islamic geometric patterns are principally derived from polygonal grids, and the Platonic and Archimedean Solids, with their polygonal surfaces, can be used as a starting point to derive such patterns.

The behavior of the regular polygons upon a sphere is radically different to their behavior in two dimensions. Any pattern which is derived from a spherical polygonal tessellation (such as the Platonic and Archimedean Solids) will have unique characteristics not found in patterns made from a two dimensional polygonal grid.



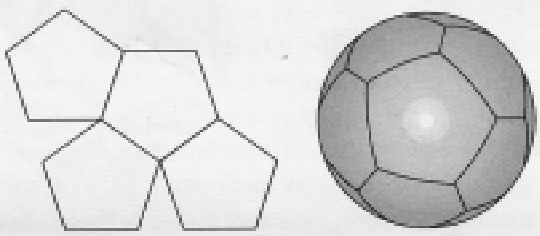
The only way to cover, or tessellate, a flat plane with equilateral triangles is for each vertex to be surrounded by six triangles. However, the equilateral triangle will cover a sphere in three ways: as a tetrahedron, where each vertex is surrounded by three triangles; an octahedron, where each vertex is surrounded by four triangles; and an icosahedron, where each vertex is surrounded by five triangles.



Spherical tetrahedron, octahedron and icosahedron. The pattern of triangles on each would be impossible on a flat plane.

Similarly, the only way to tessellate a plane with squares is for each vertex to be surrounded by four squares, resulting in the square grid familiar to all. However, the square will only cover a sphere in a single way: as a cube, where each vertex is surrounded by three squares.

As any ceramic tile maker can tell you, the regular pentagon will not tessellate on its own in two dimensions. However, on a sphere, twelve pentagons will come together to form the dodecahedron, with each of its vertices being surrounded by three pentagons.



A pentagon cannot cover a plane without leaving sinuses, but can cover the surface of a sphere to create a spherical dodecahedron.

Being that both the triangle and square will tessellate a flat plane, the geometric patterns which have been applied to the tetrahedron, octahedron, icosahedron and cube will also cover a two dimensional space. The principle difference in quality between the use of these ornamented triangles and squares on the flat plane, versus their polyhedral application, is found at the vertices. The geometric pattern on the dodecahedron model is more unique in that it will not cover a flat plane. This fivefold pattern is closely related to a variety of traditional Islamic patterns which are also comprised of a fivefold symmetry, and employ ten-pointed stars as a central feature. However, being that the dodecahedron has a *threefold* symmetry at the vertices, the pattern on this model ends up with *nine-pointed stars* at each vertex rather than the ten-pointed stars commonly found when fivefold patterns are made to cover a flat plane!

The unique effect of spherical polygonal tessellations upon geometric pattern making is even more pronounced when working with the combination of polygonal faces found in the Archimedean Solids. The geometry of these forms allows for patterns with characteristics which are impossible to find on a flat plane. For example: a close examination of the pattern designed for the Snub Cube will reveal that the stars at the vertices are *eleven-pointed!* On a two dimensional surface, a regular repeat pattern with eleven-pointed stars would be extremely unusual and difficult to execute. Other models in this collection exhibit such unusual combinations as ten- and twelve-pointed stars; eleven- and twelve-pointed stars; eight- and eleven-pointed stars; and nine- and ten-pointed stars, etc. In fact, each of the Archimedean Solids has features unique to its own geometry, and the ornamental possibilities offered by each is virtually limitless!

We hope you have fun building and examining the collection of Geodazzlers we have made for you. You are now part of a continuing tradition which started before recorded history and continues now stronger from your participation.